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Flow resistance equations without explicit estimation of the resistance coefficient for coarse-grained rivers

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Abstract

The uncertainty and subjectivity inherent in estimating the resistance coefficient is one of the main sources of error in the application of flow resistance equations to natural channels. Various studies have shown that it is possible to relate discharge successfully with variables related to the hydraulic flow geometry without the need for an independent estimate of the resistance coefficient. The aim of this paper is to calibrate and validate three models, previously proposed by other researchers, using an extensive empirical base, made up of 904 data from over 400 reaches of gravel-bed rivers and mountain streams from various regions of the world.

Thanks to the cross validation procedure, the three models were calibrated using the full database, which allowed the fitted equations to be based on the maximum number of observations. There are no important differences between the

three models calibrated with regard to their explanatory power. These models show that the exponent of the hydraulic radius is greater than 2/3 and that the exponent of the slope is closer to 1/4 than 1/2 in gravel-bed rivers and mountain streams. Validation confirmed the precision of the fitted equations, by reaching predictive power values comparable with those from calibration. The fitted equations can be successfully applied in reaches of gravel-bed rivers and mountain streams (for non-sinuous, un-vegetated and hydraulically-wide channels and for flow higher than 0.1 m³/s and lower or equal to bankfull discharge) for which there is no specific detailed information about flow resistance available.

Keywords: Gravel-bed river; Mountain stream; Fluvial hydraulics; Flow resistance

1. Introduction

The hydraulic modelling of open channels requires a flow resistance equation that allows the discharge or the flow velocity to be related to their hydraulic geometry. The equations used traditionally can be generally expressed as

$$Q = KAR^{\alpha}S^{\beta} \tag{1}$$

where Q is discharge (L^3T^{-1}), K is a coefficient, A is the cross-sectional area (L^2), R is the hydraulic radius (L), S is the friction slope, and α and β are exponents. For the Manning equation, expressed in SI units, $\alpha = 2/3$, $\beta = 1/2$ and $K = 1/n$, where n is the Manning resistance coefficient ($TL^{-1/3}$). For the Darcy-Weisbach equation $\alpha =$

1/2, $\beta = 1/2$ and $K = (8f^{-1}g)^{1/2}$, where f is the Darcy-Weisbach friction factor and g is the gravitational acceleration. Given that the value of K is not independent of the units used, throughout this paper Q is expressed as m^3/s, A in m^2, R in m and S in m/m to avoid errors.

The uncertainty that arises from estimating the K coefficient (e.g., n or f) is one of the most important sources of error in the application of this kind of equation in natural channels. The habitual procedures for estimating the resistance coefficients can be divided into the following groups: (i) resistance coefficient tables according to the type of material in the boundary and the geomorphologic characteristics of the channel (e.g., Chow, 1959; Yen, 1991); (ii) measurement of the velocity or discharge and the hydraulic geometry parameters in the reach of interest, so that the resistance coefficient in Eq. (1) can be calculated; (iii) visual photographic comparison with reaches of channels (e.g., Barnes, 1967; Hicks and Mason, 1991) for which the resistance coefficient has been calculated through the above procedure; (iv) compound formulas (e.g., Cowan, 1956; Arcement and Schneider, 1989) in which the different components of the flow resistance are separated lineally, estimating these components by means of procedures similar to those described in (i) and (iii); (v) semi-empirical formulas, where distinctions must be made between those that can be applied to alluvial channels with granular boundary and those for vegetated channels (Yen, 1991; Yen, 2002). In the first case, the resistance coefficient (e.g., f) is a function of the relative submergence (i.e., y/d_i, where y is the mean water depth and

d_i a characteristic grain-size) and additionally, in the case of the presence of bed-forms in sand-beds, of other variables such as the Froude number (Fr) or the particle Froude number (Fr_*). In the case of vegetated channels, the derived equations depend on variables related to the geometry, flexibility, relative submergence and distribution density of the vegetation.

Procedures (i), (iii) and (iv) include a high degree of uncertainty and, to a varying degree, are subjective. Thus, the estimated values can vary considerably, even when these procedures are applied by experts (Burnham and Davis, 1990). Procedure (ii) is not always possible, either because of the cost it represents, lack of time or because the discharge or range of discharges of interest have a high return period or are of catastrophic nature. Procedure (v) has the advantage of eliminating the subjectivity although, in contrast, it obliges one to have precise information about the characteristics of the sediment, especially the size distribution, and the vegetation cover.

Given the inherent difficulties for estimating the resistance coefficient, various researchers have proposed alternative approaches with the aim of avoiding the explicit estimation of this coefficient. One of the first options is that of correlating n or f with some of the independent variables from Eq. (1), generally S and R. Table 1 shows some of the equations obtained by this procedure: (2) (Golubtsov, 1969), (7), (9) (Bray, 1979), (11) (Jarrett, 1984) and (14) (Sauer, 1990).

Table 1. Equations derived for natural channels (SI Units)

Reference	Equation	Nº	Q (m^3/s)	A (m^2)	R (m)	S (%)	N_c	d_{50} (m)
Golubtsov (1969)	$Q = 4.50AR^{2/3}S^{1/6}$	(2)	—	—	—	0.4–20	500	—
	$(n = 0.222S^{1/3})$	(3)						
Riggs (1976)	$Q = 1.55A^{1.33}S^{0.05-0.056\log S}$	(4)	—	—	—	—	62	—
Williams (1978)	$Q = 4.0A^{1.21}S^{0.28}$	(5)	0.5–28.3 · 10^3	0.7–8.51 · 10^3	0.25–16.7	0.0041–8.1	233	0.00019–0.19
Bray (1979)	$Q = 7.96AR^{0.60}S^{0.29}$	(6)	5.52–8.21 · 10^3	6.33–3.73 · 10^3	0.44–6.92	0.022–1.5	67	0.019–0.145
	$Q = 9.62AR^{2/3}S^{0.32}$	(7)						
	$(n = 0.104S^{0.177})$	(8)						
	$Q = 6.17AR^{\frac{1}{2}}S^{0.24}$	(9)						
	$(\sqrt{8/f} = 1.97S^{-0.26})$	(10)						
Jarrett (1984)	$Q = 3.17AR^{0.83}S^{0.12}$	(11)	0.34–128.2	1.03–63.4	0.15–2.1	0.2–4.0	75	0.06–0.43
	$(n = 0.32S^{0.38}R^{-0.16})$	(12)						
Meunier (1989)	$Q = 1.3AR^{0.86}S^{-0.084}$	(13)	0.137–195	0.52–79.5	0.102–1.60	0.4–4.0	44	0.06–0.34
Sauer (1990)	$Q = 8.33AR^{0.59}S^{0.32}$	(14)	—	—	< 5.8	0.03–1.8	—	—
	$(n = 0.12S^{0.18}R^{0.08})$	(15)						
Dingman and Sharma (1997)	$Q = 1.56A^{1.17}R^{0.40}S^{-0.0543\log S}$	(16)	0.01–11.5 · 10^3	0.45–4.51 · 10^3	0.11–9.17	0.002–4.18	520	—
Bjerklie et al. (2003)	$Q = 7.22A^{1.02}R^{0.72}S^{0.35}$	(17)	0.01–216 · 10^3	0.52–109 · 10^3	0.18–48	7 · 10^{-5}–4.0	506	—
Bjerklie et al. (2005)	$Q = 7.14AR^{0.67}S^{0.33}$	(18)	0.01–27.6 · 10^3	0.29–12.5 · 10^3	0.1–12.39	0.0043–4.0	680	—
	$Q = 4.84A^{1.10}R^{0.53}S^{0.33}$	(19)						

N_c = number of calibration data. In equations (13), (17), (18) and (19) it was supposed that $y = R$.

Another option is to fit the parameters of Eq. (1) (or of the equivalent equation that takes the cross sectional mean flow velocity as a dependent variable) directly through regression, as is the case in Eqs. (6) (Bray, 1979) and (13) (Meunier, 1989) (Table 1). Moreover, by means of multiple regression analysis, some authors have derived equations that vary with regard to the model represented by Eq. (1) (Table 1). For example, Eqs. (4) (Riggs, 1976) and (5) (Williams, 1978) do not include R as an independent variable and adopt an exponent of A different from 1.0. Similarly, in Eqs. (16) (Dingman and Sharma, 1997), (17) (Bjerklie et al., 2003) and (19) (Bjerklie et al., 2005), the exponent of A was taken as a fitting parameter. Furthermore, Riggs (1976) (Eq. (4)) and Dingman and Sharma (1997) (Eq. (16)) found that the predictive power of the fitted model increased if the exponent of S was a logarithmic function of S.

On the other hand, from the value of the exponent of S obtained in Eq. (17) and in a simplified version of Eq. (16), together with theoretical considerations, Bjerklie et al. (2005) sustain that this exponent is closer to 1/3 than 1/2 in natural channels (this latter value corresponding to the Manning and Darcy-Weisbach formulas). Accordingly, Eq. (18) was fitted imposing 0.67 as a value of the exponent of R and 0.33 as a value for that of S, which is equivalent to substituting a correlation of the $n = aS^{0.17}$ type in the Manning equation (where a is the fitting coefficient). The predictive power of Eqs. (18) and (19), calibrated and validated with the same databases, was very similar.

Considering the equations obtained previously by different authors, the following models were adopted for evaluation in this paper

$$Q = K_1 A R^{\alpha_1} S^{\beta_1} \tag{20}$$

$$Q = K_2 A^{\delta_2} R^{\alpha_2} S^{\beta_2} \tag{21}$$

$$Q = K_3 A^{\delta_3} R^{\alpha_3} S^{\beta_3 \log S} \tag{22}$$

where K_i, α_i, β_i and δ_i represent fitted parameters.

However, if constant value of K is adopted to model a very wide range of discharges or for channels with very varied geomorphologic characteristics (boulder, gravel or sand-bed rivers, vegetated channels, etc.) it should be expected that the result will be less satisfactory than if narrower discharge intervals are adopted (Bjerklie et al., 2003) or more homogenous geomorphological characteristics are imposed. For example, with reference to flow resistance, a distinction can be made between alluvial plain sand-bed rivers and coarse material bed rivers, given that, among other differences, the latter have coarser and more heterogeneous sediment, steeper slopes, lower relative submergence and large grain and form roughness (Bathurst 1993, Wohl, 2000), which would affect the value of K.

The aim of this paper is to evaluate models (20), (21) and (22) (which covers calibration, validation and comparison), using a very extensive database for gravel-bed rivers and mountain streams and representing a wide hydraulic and geomorphological range. The restriction to gravel-bed rivers and mountain streams also means an indirect narrowing of the discharge range. Both of these characteristics contribute to achieving a higher predictive power when a constant value is adopted for K. Owing to the type of data selected, as well as allowing the traditional application of fitted equations to bankfull or flood levels in rivers, this also responds to the need for hydraulic modelling of small streams with low submergence, which is of use in fluvial restoration or ecological studies. On the other hand, by applying the cross validation technique, the aim is to obtain a notably bigger calibration database than those used in the past for developing the models currently available (Table 1).

In resume, the aim is to propose a set of models (based on variables which are easily measured in the field) that are applicable in reaches of interest of gravel-bed rivers and mountain streams for which enough information about the flow resistance is not available.

2. Material and methods

The selection criteria adopted to compile the database for calibration and validation are presented below. The river reach must correspond to a single, approximately straight, channel, along which there is a steady and macroscopically uniform flow. It must be free of vegetation and obstacles (natural or artificial). Such conditions allow us to assume that the effects of vegetation and changes in the shape of the channel (cross section, slope and alignment) on flow resistance are minimal. However, it must be taken into account that, owing to the channel morphology of mountain and gravel-bed rivers and their low relative submergence conditions, in reality, the flow is varied on a detailed scale. Thus, the requirement of uniformity must be understood as the mean along the reach, that is, it is considered sufficient for the flow to be macroscopically uniform. Moreover, it has been confirmed that flow is turbulent (Reynolds number (Re) of over 2,000) and hydraulically rough (grain shear Reynolds number (Re_*) of over 200). The ratio of the free surface width (T) to the cross-sectional mean depth (y) must be higher than approximately 10 (which implies that $R \approx y$), with the aim of ensuring that the flow in the central zone of the cross section is not influenced by the channel banks. The discharge of the selected data must, in all cases, correspond to in-bank level. Moreover, the sediment must be gravel, cobble or boulder size, explicitly excluding channels with beds made up of cohesive sediment, sand or rock. For this reason, the value of d_{50} (the size of the median axis of the bed material for which 50% of the material is finer) has been set at greater than or equal to 2 mm.

Applying the above-mentioned requirements produced a set of 904 data from rivers, corresponding to 24 bibliographic references from the 1955-2002 period and also to measurements derived from our own research in rivers on the Spanish side of the Pyrenees range (López, 2005) (see Appendix A). This database is made up of over 400 reaches of various gravel-bed rivers and mountain streams, mainly located in the United States, New Zealand, Canada and the United Kingdom. The hydraulic geometry variables of each reach (A, R and S) were determined by measuring different cross sections (generally, three or more) separated by a distance equivalent to several times the channel width, so the data included represented the mean characteristics at the reach scale and did not reflect the conditions in a single section. In some reaches, measurements were taken for more than one flow rate.

Table 2 shows the minimum and maximum values, the mean and the coefficient of variation of Q, A, R, S and d_{50} concerning the calibration database. As can be seen, the selected set is representative of a wide geomorphologic and hydraulic range in the context of coarse-grained channels. The discharge varies by six orders of magnitude, the cross-sectional area by five, the hydraulic radius by two, the slope by four and the median diameter by two.

Table 2. Range of hydraulic variables in the calibration and validation databases

Parameter	Symbol	Units	Minimum	Maximum	Mean	C_v (%)
Calibration Database (N = 904)						
Discharge	Q	m^3/s	$3.50 \cdot 10^{-3}$	$8.21 \cdot 10^{3}$	$9.22 \cdot 10^{1}$	496
Cross-sectional area	A	m^2	0.05	3,737	46.73	430
Hydraulic radius	R	m	0.03	6.92	0.80	105
Bed slope	S	m/m	$1.00 \cdot 10^{-5}$	$1.60 \cdot 10^{-1}$	$1.09 \cdot 10^{-2}$	146
Median grain-size	d_{50}	m	0.007	0.51	0.11	88
Validation Database V_1 (N = 452)						
Discharge	Q	m^3/s	$3.50 \cdot 10^{-3}$	$8.21 \cdot 10^{3}$	$9.72 \cdot 10^{1}$	525
Cross-sectional area	A	m^2	0.07	3,262	48.94	425
Hydraulic radius	R	m	0.05	6.92	0.81	102
Bed slope	S	m/m	$1.00 \cdot 10^{-5}$	$1.25 \cdot 10^{-1}$	$1.04 \cdot 10^{-2}$	138
Median grain-size	d_{50}	m	0.010	0.51	0.11	86
Validation Database V_2 (N = 452)						
Discharge	Q	m^3/s	$4.00 \cdot 10^{-3}$	$7.22 \cdot 10^{3}$	$8.72 \cdot 10^{1}$	458
Cross-sectional area	A	m^2	0.05	3,737	44.51	436
Hydraulic radius	R	m	0.03	6.83	0.79	108
Bed slope	S	m/m	$8.50 \cdot 10^{-5}$	$1.60 \cdot 10^{-1}$	$1.14 \cdot 10^{-2}$	153
Median grain-size	d_{50}	m	0.007	0.51	0.11	92

In this study, the evaluation of models covers three levels: calibration, validation and comparison of the models. The calibration consisted of fitting the parameters of the selected models, by means of the least-squares procedure, using the full database ($N = 904$). Given that the least-squares procedure tends to minimise the residuals corresponding to the data with the greatest magnitude, the models fitted by means of this procedure can commit large errors of prediction for small magnitude Q values. The above is more marked when the variability studied is expanded to a wide range, as in this case. That is why the parameters of the logarithmic transformation of models (20), (21) and (22) were fitted by means of the least squares method, that is with the following equations

$$\log Q = \log K_1 + \log A + \alpha_1 \log R + \beta_1 \log S \tag{23}$$

$$\log Q = \log K_2 + \delta_2 \log A + \alpha_2 \log R + \beta_2 \log S \tag{24}$$

$$\log Q = \log K_3 + \delta_3 \log A + \alpha_3 \log R + \beta_3 \log^2 S \tag{25}$$

With the aim of evaluating the goodness-of-fit of the selected models, the following statistics were calculated: standard error of estimate (SE); coefficient of determination (R^2); modified coefficient of efficiency (E'); mean relative error (MRE); percentage of data with a relative error less than, or equal to 50% (RE_{50}) and 25% (RE_{25}); mean symmetry error (MSE); percentage of data with overestimate error (OE). The SE statistic is the square root residual mean square

$$\mathrm{SE} = \sqrt{\frac{\sum_{i=1}^{N} (O_i - P_i)^2}{N-1}} \tag{26}$$

where O_i is the observed i value of the variable, P_i is the predicted i value of the variable and N is the number of data. R^2 is the square of Pearson's product-moment correlation coefficient, and can be interpreted as the proportion of the total variance in the observed data that can be explained by the model. It has been calculated as

$$R^2 = \left(\frac{\sum_{i=1}^{N} (P_i - \overline{P})(O_i - \overline{O})}{\sqrt{\sum_{i=1}^{N} (P_i - \overline{P})^2} \sqrt{\sum_{i=1}^{N} (O_i - \overline{O})^2}} \right)^2 \tag{27}$$

where $\overline{P}$ and $\overline{O}$ are the average of respectively the predicted and observed N values of the variable. The statistic E' is intended to reduce the magnifying effect that causes the error in high magnitude data and which introduce the squared terms, so it is based on the absolute value of the residuals (Legates and McCabe, 1999)

$$E' = 1.0 - \frac{\sum_{i=1}^{N} |O_i - P_i|}{\sum_{i=1}^{N} |O_i - \overline{O}|} \tag{28}$$

The MRE statistic is the mean percentage error between observation and prediction with regard to the observed value

$$\mathrm{MRE} = \frac{100}{N} \sum_{i=1}^{N} \frac{|P_i - O_i|}{O_i} \tag{29}$$

so the error in each data contributes equally to the total value, independently of its magnitude. The MSE statistic was calculated as

$$\mathrm{MSE} = \frac{100}{N} \sum_{i=1}^{N} \frac{P_i - O_i}{O_i} \tag{30}$$

so this represents a measurement of the prediction symmetry with respect to the line of perfect agreement. Similarly, the relation between the observed and predicted values of the dependent variable was analysed graphically for each data. It must be noted that the fitted statistics referred to above were calculated for the antilogarithmic version of the expression resulting from the calibration, this being Eqs. (20), (21) and (22). This is justified because the user of the fitted models normally applies them in their antilogarithmic version, which is the common way of presenting them, so the statistical assessment of this version is more interesting.

In the second phase, the models were validated by means of cross validation in its test set switch modality (Esbensen et al., 1994). To this end, the full database (N = 904), previously used for calibration, was randomly divided into two sets (V_1 and V_2) each made up of 50% of the total number of data (i.e., $N = 452$). Later, and independently for each validation set (V_1 and V_2), the fitting phase was repeated for the three models evaluated, thus obtaining two equations for each model. Lastly, for the equation fitted with the set of data V_1, the goodness-of-fit indices were calculated but using the data from set V_2, and vice versa. The final value of the statistical validation indices was obtained as the average of those obtained in each validation set. This aims to avoid the loss of information in the calibration set that the test set validation would involve (owing to the splitting of the database); at the same time as, thanks to the cross validation, a measure is also available of the prediction error committed by the fitted equations when these are applied to independent cases.

In the third phase, the three fitted models were compared with each other, taking as a criteria the value of the statistics presented above, both the result of calibration and validation. Moreover, note was taken of the value of the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) statistics

$$\text{AIC} = N \ln \left(\frac{\sum_{i=1}^{N} (O_i - P_i)^2}{N - k - 1} \right) + 2(k+1) \tag{31}$$

$$\text{BIC} = N \ln \left(\frac{\sum_{i=1}^{N} (O_i - P_i)^2}{N - k - 1} \right) + k\, ln\, N \tag{32}$$

where k is the number of parameters in the model. These indices are frequently used to select the best model when more than one model is evaluated with the same database.

3. Results and discussion

Table 3 shows Eqs. (33), (34) and (35), antilogarithmic versions of the equations fitted in accordance with models (23), (24) and (25) respectively. Moreover, Table 3 specifies the values of the fitted statistics with regard to the calibration database ($N = 904$), corresponding to Eqs. (33), (34) and (35). On the other hand, Fig. 1 shows the observed discharge against that predicted by Eqs. (33), (34) and (35). Table 4 shows the 95% confidence interval for the fitted parameters in Eqs. (33), (34) and (35).

Table 3. Fitted equations and statistics of calibration and validation

Statistic	Eq. (33)		Eq. (34)		Eq. (35)	
	$Q = 6.04AR^{0.82}S^{0.26}$		$Q = 5.56A^{1.03}R^{0.77}S^{0.27}$		$Q = 2.93A^{1.02}R^{0.79}S^{-0.057\log S}$	
	Calibration	Validation	Calibration	Validation	Calibration	Validation
SE (m^3/s)	318	294	349	327	310	276
R^2	0.920	0.962	0.917	0.959	0.901	0.960
E'	0.769	0.764	0.754	0.750	0.791	0.788
MRE (%)	35.4	35.6	35.5	35.4	33.1	33.1
RE_{25} (%)	52.8	51.7	52.3	51.4	53.5	53.7
RE_{50} (%)	80.9	80.4	80.0	79.8	81.9	81.5
MSE (%)	10.1	10.5	10.4	10.3	9.1	9.5
OE (%)	47	51	47	51	48	51
AIC	10 432		10 606		10 391	
BIC	10 444		10 623		10 408	

Table 4. Confidence interval for the parameters of Eqs. (33), (34) and (35)

Confidence interval	Parameter										
	Eq. (33)			Eq. (34)				Eq. (35)			
	K_1	α_1	β_1	K_2	δ_2	α_2	β_2	K_3	δ_3	α_3	β_3
Lower 95%	5.27	0.79	0.24	4.63	0.98	0.67	0.24	2.51	0.98	0.70	-0.062
Upper 95%	6.93	0.86	0.29	6.67	1.08	0.86	0.29	3.43	1.07	0.88	-0.053

At the 95% confidence level there are no significant differences between the coefficients of Eqs. (33) and (34), the most directly comparable ones. Similarly, the value of the exponents of A and R in Eq. (35) is not significantly different, at the 95% confidence level, from the value of these exponents in Eqs. (33) and (34). Although the value of the exponent of A (Eqs. (34) and (35)) does not differ significantly from 1.0, the exponents of R and S (Eq. (33) and (34)) do differ at the 95% confidence level from the values of these exponents in the Manning and Darcy-

Weisbach equations. However, note that the lower end of the confidence interval of the exponent of R in Eq. (34) is exactly equal to 0.67.

The three calibrated models show a similar explanatory power, as is deduced from comparing the value of the fitting statistics. Nevertheless, in accordance with the majority of the statistics considered, Eq. (35) is the one that behaves best, followed by Eq. (33) and, in last place, Eq. (34). The only exception to the above-mentioned sequence is R^2, for which Eq. (33) is the best and Eq. (35), the worst, although the difference between the two is only 2%.

The OE value for the three fitted models indicates that the proportion of data for which the discharge was underestimated is slightly higher. However, according to the MSE value, the average of the deviations is above the line of the perfect agreement. This means that, although there were proportionally fewer overestimated data, on average these had a higher relative error.

As can be seen in Table 3, the MRE value for the three fitted models ranges between 33 and 36%. The total value of MRE (Eq. (29)) can be attributed to various kinds of error: measurement error of the independent variables in the prediction model (i.e., A, R and S), error through non-completion of the hydraulic and geometrical hypothesis in the reaches observed and, lastly, error derived from the model's lack of explanatory power. If Eq. (33) is taken as a reference (given that it occupies an intermediate position with regard to the MRE value) and it is assumed that $y = R$ and knowing that $y = AT$, the prediction error of Q ($\in Q$) obtained by aggregating the measurement errors (Bathurst, 1986) can be estimated as

$$\in Q = \pm((\in T)^2 + (1.82 \in y)^2 + (0.26 \in S)^2)^{1/2} \tag{36}$$

where $\in T$, $\in y$ and $\in S$ represent the measurement errors of T, y and S, respectively. If it is estimates that: $\in T = 1\%$, $\in y = 5\%$ and $\in S = 8\%$ (Bathurst, 1986), then $\in Q = \pm 9.4\%$ (a similar value to the one that would be obtained from Eqs. (33) and (35)). For comparison purposes, note that the estimation error of the Manning coefficient through tables or photographs is approximately 25%. The error for non-completion of the hydraulic and geometrical hypotheses is difficult to estimate given the high number of data sources consulted. Thus, the error that is attributed to lack of explanatory power of the fitted models in terms of MRE is estimated at a maximum of 24–27%. In any case, it must be emphasised that throughout this analysis the measurement error of the observed discharge has not been taken into consideration.

Although the models evaluated were transformed logarithmically to fit them, Fig. 1 shows that Eqs. (33), (34) and (35) tend towards a strong overestimation in the lower discharge range ($\approx< 0.1$ m^3/s). This can be attributed to the fact that the explanatory power of the models that adopt a constant value for K diminishes when the discharge is extended over a wide range. Thus, it is recommended to avoid the use of the models fitted in this paper when the observed discharge is less than 0.1 m^3/s (or when the predicted discharge is less than 0.2 m^3/s). Other authors have also detected a strong overestimation for low discharges. Dingman and Sharma (1997) warned that Eq. (16) did not give reliable predictions if the discharge was less than 3 m^3/s. The validation of Eqs. (17) (Bjerklie et al., 2003), (18) and (19) (Bjerklie et al., 2005), reveals that these equations tend to overpredict acutely for discharges of less than 1 or 2 m^3/s. As shown above (Bjerklie et al., 2003), better results in this respect would presumably be reached if the equations had been fitted by splitting the database into more specific discharge ranges. In any case, the recommended lower limit of Q for the application of the models calibrated in this paper is significantly lower than that detected by the authors cited above, which places these models at an advantage when studying smaller and shallower rivers.

It is also possible that better results would have been obtained if the data selection had been based on even more homogeneous geomorphologic criteria than those adopted in this paper. For example, a distinction could have been made between gravel-bed rivers, on one hand, and cobble-and boulder-bed streams, on the other. Effectively, the above has consequences not only for the average particle size, but also the characteristic bedforms, mean slope and relative submergence, given that all these variables are interrelated. Frequently, channel slope has been adopted as a control variable for segmenting the database. The threshold values found by different authors generally vary between 0.5% and 1%. For example, both Rickenmann (1994) and Bathurst (2002) coincided in a value of 0.8%, on detecting differentiated tendencies in the behaviour of flow resistance around this value. However, Bathurst (2002) found that these differentiated tendencies could not be explained by the relative differences in bedforms in the selected data, so he proposed detailed studies into the dependence of flow resistance on the velocity profile, wave drag from protruding boulders or the bed material characteristics.

An alternative procedure to reduce the limitations derived from taking K as a constant would be to find satisfactory correlations with variables that are easily determined in the field. For example, Bjerklie et al. (2005) suggested parameters such as the width and the ratio of the cross-sectional flow area to the bankfull channel cross-sectional area.

In Eqs. (33) and (34), it can be seen that the exponent of R is above 2/3 and that the exponent of S is lower than 1/2, which are the values corresponding to the Manning formula. The tendency observed coincides with that in Eq. (11), although in this the exponent of S is much lower, and Eq. (17), the latter being the most similar to Eqs. (33) and (34) among the equations that appear in Table 1. Thus, coinciding with Bjerklie et al. (2005), in the case of natural channels, the exponent of S is closer to 1/3 than to 1/2, although in the fitted models in this paper it is even slightly lower, given that it approaches 1/4. In fact, Table 4 shows that the exponent of S in Eqs. (33) and (34) is significantly different from 1/3 at the 95% confidence level.

Table 2 shows the minimum and maximum values, the mean and the coefficient of variation of Q, A, R, S and d_{50} for the two randomly generated validation sets (V_1 and V_2). It can be seen that the two validation sets are very similar to each other and also resemble the calibration database. Moreover, test F for equality of variances and test t for equality of means of two independent samples applied to sets V_1 and V_2 for each of the variables shown in Table 2 allow the equality of both parameters to be accepted for a significance level clearly higher than 0.05.

Table 3 shows the value of the different statistical indices resulting from the cross validation. If these are compared with those corresponding to the calibration, it can be deduced that there is practically no difference, which confirms the precision of the three fitted models. The validation confirms, similarly, the calibration results so it respects the comparison of the explanatory power of the fitted models.

4. Conclusions

The three fitted models are valid for hydraulically-wide channels of gravel-bed rivers and mountain streams in which the effects of vegetation and changes in channel shape on the flow resistance are minimum. Thanks to the cross validation method, the calibration database is made up of a total of 904 data, one of the largest used to date for fitting models of flow resistance in gravel-bed rivers.

The three fitted models are very similar with regard to their goodness-of-fit and predictive power. However, in accordance with the findings of other authors, the model in which the exponent of S is a logarithmic function of S (Eq. (35)) offers a slightly better result than the other two. If these (Eqs. (33) and (34)) are compared with the Manning equation, it is deduced that the exponent of S in gravel-bed rivers and mountain streams is closer to 1/4 than to 1/2 and that the exponent of R is closer to 4/5 than to 2/3.

In contrast with models that require a more or less subjective estimation of the resistance coefficient, the fitted models in this paper can be usefully applied to reaches of gravel-bed rivers and mountain streams for which there is no specific and detailed information about the flow resistance, or even when this information is available but must be used by non-expert personnel. The fitted models are applicable to in-bank discharges, from the bankfull level to conditions of low relative submergence. The mean relative error of the three models ranges between 33 and 36%, it being estimated that approximately 9% can be attributed to measurement errors in the model's independent variables. Owing to the prediction error, the application of the fitted models is not recommended when the discharge is lower than 0.1–0.2 m^3/s. However, this limit is lower than that detected for models derived by other authors, which oscillate between 1 and 3 m^3/s. This places the models fitted in this paper at an advantage in, for example, the context of ecological studies or fluvial restoration in smaller and shallower rivers.

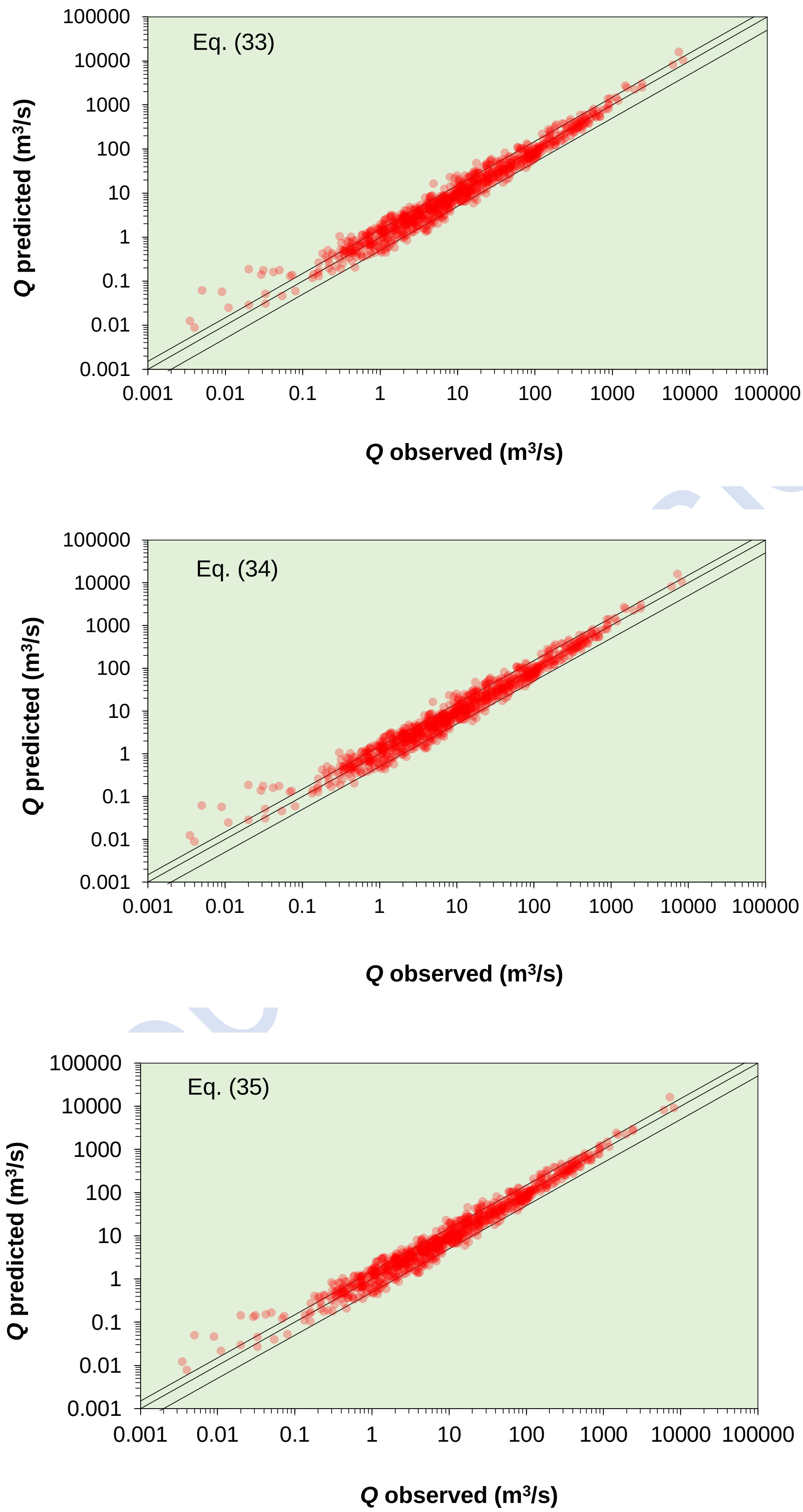

Fig 1. Predicted discharge plotted against observed discharge for the calibration data set using equations (33), (34), and (35). The lines corresponding to perfect agreement and ±50% error are shown.

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Appendix A

This appendix cites the bibliographic sources consulted to make up the study database together with the number of data selected from each reference.

Andrews (1984) ($N = 14$); Bathurst (1978) ($N = 9$); Bathurst (1985) ($N = 44$); Barnes (1967) ($N = 14$); Bray (1979) ($N = 67$); Charlton et al. (1978) ($N = 12$); Colosimo et al. (1988) ($N = 43$); Crusellas (2000) ($N = 24$); Griffiths (1981) ($N = 136$); Hey (1979) ($N = 30$); Hey and Thorne (1986) ($N = 10$); Hicks and Mason (1991) ($N = 99$); Jarrett (1984) ($N = 66$); Judd and Peterson (1969) ($N = 116$); Kellerhals (1967) ($N = 22$); Lee and Ferguson (2002) ($N = 18$); López (2005) ($N = 16$); Marcus et al. (1994) ($N = 15$); Maresova and Mares (1989) ($N = 74$); Nikora et al. (1998) ($N = 6$); Pitlick (1992) ($N = 9$); Prestegaard (1983) ($N = 6$); Thompson and Campbell (1979) ($N = 5$); Thorne and Zevenbergen (1985) ($N = 12$); Wolman (1955) ($N = 37$).